Radiative Dileptonic Decays of B Mesons

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Abstract

We investigate the radiative dileptonic decays $B_s(B_d) \to \gamma l^+ l^-$ within the standard model. Using the constituent quark model, the branching ratios turn out to be around 5×10^{-9} for $B_s \to \gamma \mu^+ \mu^-$ and around 6×10^{-10} for $B_d \to \gamma \mu^+ \mu^-$, with slightly larger values for $B_s(B_d) \to \gamma e^+ e^-$. The differential rate as a function of the dilepton invariant mass is given. The possibility of using these processes to determine the decay constants of B_s or B_d is discussed.

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1 Introduction

Rare decays induced by flavor changing neutral currents can be used as tests of the Standard Model (SM), and are sensitive to new physics. It is expected that at future B factories and fixed target machines, other rare decay channels of the bottom quark [1] will be discovered in addition to the observed $b \to s\gamma$ transition[2]. These processes also offer useful information for extracting the fundamental parameters of the SM, such as $|V_{ub}|$ [3].

Rare decays can also serve as alternative channels to measure some elementary hadronic parameters. For instance, the decay constants f_{B_q}, $q = s, d$ can be extracted from $B_q \to \gamma\nu\bar{\nu}$ [4]. In the present work, we investigate the possibility of using the channel, $B_q \to \gamma l^+ l^-$ to determine the decay constants. Pure leptonic decays of heavy pseudoscalar mesons into light lepton pairs are helicity suppressed, their branching ratios are of the order of 10^{-9} for $B_s \to \mu^+\mu^-$, and 10^{-14} for $B_s \to e^+e^-$ [5]. This makes it difficult to determine f_{B_s} from these processes. For B_d the situation is even worse due to the smaller CKM angle. If a photon is emitted in addition to the lepton pair, then the mechanism of helicity suppression will not hold any longer and larger branching ratios are expected. Although the process $B_s \to \tau^+\tau^-$ is free from helicity suppression and its branching ratio is around 8×10^{-7} in the SM [6], it is likely to be compatible with the decays into lepton pairs only when its efficiency is better than 10^{-2}.

In Section 2 the relevant effective Hamiltonian will be given in the SM, and the constituent quark model will be used to give the numerical predictions. Finally, we make some comments in Section 3.

2 Model calculations

The most important contributions to $B_q \to \gamma l^+ l^-$ $(l = e, \mu)$ stem from the effective Hamiltonian which induces the pure leptonic processes $B_q \to l^+ l^-$. Let us start with the short

distance contributions to the latter one, which include box, Z and photon penguin diagrams. The Feynman diagrams are displayed in Fig.1. The QCD corrected effective Hamiltonian in the SM is [7, 8]:

$$\mathcal{H}_{eff} = \frac{\alpha G_F}{\sqrt{2}\pi} V_{tb} V_{tq}^* \left[\left(C_9^{eff} \bar{q}\gamma^\mu P_L b + \frac{2C_7 m_b}{p^2} \bar{q} \not{p}\gamma^\mu P_R b \right) \bar{l}\gamma_\mu l + C_{10}(\bar{q}\gamma^\mu P_L b)\bar{l}\gamma_\mu\gamma_5 l \right], \quad (1)$$

with $P_L = (1 - \gamma_5)/2$, $P_R = (1 + \gamma_5)/2$, and $p = p_+ + p_-$ is the momentum of the lepton pair. The QCD corrected Wilson coefficients $C_7(m_b)$, $C_9^{eff}(m_b)$, and $C_{10}(m_b)$ are given by Misiak [8].

Because of the lightness of the leptons e and μ, the pure leptonic processes $B_q \to l^+ l^-$ are suppressed by helicity. If a photon line is attached to any of the charged lines in Fig.1, the situation will be different: helicity suppression is overcome. When the photon is attached to internal lines, there will be a suppression factor of m_b^2/M_W^2 in the Wilson coefficient, since the resulting operators(dimension-8) are two orders higher in dimensions than the usual ones(dimension-6).

One may expect that the additional two diagrams displayed in Fig.2 (a) and (b) might also contribute to the decay $B_q \to \gamma l^+ l^-$. However, our calculation shows that the contribution of Fig.2(b) is quite small and its influence can be neglected in the numerical results. Fig.2(a) involves a singularity, since the intermediate s quark can be on its mass shell. The influence of this singularity is an artifact of the constituent quark model employed here. If we calculate the contribution of Fig. 2(a) to the differential decay rate anywhere except near the singularity we find it to be negligible (at most a few percent). Therefore we neglect it altogether.

There are also long distance contributions to $B_q \to l^+ l^- \gamma$. For instance, in the B_s decay, there are cascade processes at the hadronic level:

$$B_s \to \phi(\phi', J/\psi...)\gamma, \quad \phi(\phi', J/\psi...) \to l^+ l^-. \quad (2)$$

These involve the on-shell $\phi, \phi', J/\psi...$ and are similar to $b \to sJ/\psi \to sl^+l^-$ [11]. They are estimated to be of order of 10^{-9}, which is about the same order of the short distance

contribution. All these long distance effects, which are shown as sharp peaks in the invariant mass spectrum of the lepton pair, will not be included in the following.

Now using a simple constituent quark model (see, for example, [9]), we calculate the amplitude for diagrams with photon emitted from external fermion lines. First, for diagrams with photon attached to external lepton lines, we get the amplitude proportional to:

$$\bar{q}\gamma_\mu b_L \epsilon_\nu \bar{l} \left[\gamma^\nu \frac{\not{p}_- + \not{p}_\gamma + m_l}{(p_- + p_\gamma)^2 - m_l^2} \gamma^\mu (C_9^{eff} + C_{10}\gamma_5) - \gamma^\mu (C_9^{eff} + C_{10}\gamma_5) \frac{\not{p}_+ + \not{p}_\gamma - m_l}{(p_+ + p_\gamma)^2 - m_l^2} \gamma^\nu \right] l. \tag{3}$$

Using the decay constant definition:

$$< 0|\bar{q}\gamma^\mu \gamma_5 b|B > = -f_{B_q} p_B^\mu, \tag{4}$$

we can easily calculate the results of eqn.(3). Note that, there is no contribution from the operator O_7 in this circumstance, since the definition of the decay constant shows that:

$$< 0|\bar{q}\sigma^{\mu\nu} P_R b|B > = 0. \tag{5}$$

For the other two diagrams with photon emitted from the external quark lines (b or q), the amplitude is proportional to:

$$\epsilon_\nu \bar{q} \left[\gamma^\nu \frac{\not{p}_\gamma - \not{p}_q + m_q}{(p_q \cdot p_\gamma)} \gamma^\mu P_L + P_R \gamma^\mu \frac{\not{p}_b - \not{p}_\gamma + m_b}{(p_b \cdot p_\gamma)} \gamma^\nu \right] b \left[C_9^{eff} \bar{l}\gamma_\mu l + C_{10}\bar{l}\gamma_\mu \gamma_5 l) \right]$$

$$+ 2\frac{C_7 m_b}{p^2} \epsilon_\nu \bar{q} \left[P_R \not{p}\gamma^\mu \frac{\not{p}_b - \not{p}_\gamma + m_b}{(p_b \cdot p_\gamma)} \gamma^\nu + \gamma^\nu \frac{\not{p}_\gamma - \not{p}_q + m_q}{(p_q \cdot p_\gamma)} \not{p}\gamma^\mu P_R \right] b \, (\bar{l}\gamma_\mu l). \tag{6}$$

In the constituent quark model, $p_q^\mu = (m_q/m_B)p_{B_q}^\mu$, and $p_b^\mu = (m_b/m_B)p_{B_q}^\mu$. Then applying eqn.(4), neglecting terms suppressed by m_q/m_b, $q = d, s$, we get:

$$\mathcal{A} = C \left[i\epsilon_{\alpha\beta\mu\nu}\epsilon_\gamma^\alpha p_\gamma^\beta p_{B_q}^\nu + (p_{\gamma\mu}\epsilon_{\gamma\nu} - p_{\gamma\nu}\epsilon_{\gamma\mu})p_{B_q}^\nu \right] \left[\left(C_9^{eff} - \frac{2C_7 m_{B_q}^2}{p^2} \right) \bar{l}\gamma^\mu l + C_{10}\bar{l}\gamma^\mu \gamma_5 l \right], \tag{7}$$

with

$$C \equiv \frac{e\alpha G_F f_{B_q} m_{B_q}}{12\sqrt{2}\pi m_q (p_{B_q} \cdot p_\gamma)} V_{tb}V_{tq}^*.$$

After squaring the total decay amplitude, we find that the contribution of diagrams with photon attached to the lepton lines (eqn.(3)) is proportional to m_l, which is negligible in

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numerical calculations. The main contribution to $B_q \to \gamma l^+ l^-$ is thus from the square of eqn.(7). Performing the phase space integration over one of the two Dalitz variables, we get the differential decay width versus $s = (p_+ + p_-)^2/m_{B_q}^2$:

$$\frac{d\Gamma}{ds} = \frac{\alpha^3 G_F^2 f_{B_q}^2 m_{B_q}^5}{3456\pi^4 m_q^2} |V_{tb}V_{tq}^*|^2 (1-s)s \left(C_{10}^2 + \left| C_9^{eff} - \frac{2}{s}C_7 \right|^2 \right). \tag{8}$$

We see that the differential rate is proportional to $(f_{B_q}/m_q)^2$. Using $\alpha = 1/137$, $m_s = 0.51$ GeV, $m_t = 176$ GeV and $|V_{tb}V_{ts}^*| = 0.04$, the differential decay rate as a function of s is displayed in Fig.3, which shows that the contribution from soft photons, corresponding to large s region, is negligibly small.

After integration, we also get:

$$\Gamma(B_s \to \gamma e^+ e^-) = 3.0 \times 10^{-21} \times \left(\frac{f_{B_s}}{0.2\text{GeV}} \right)^2 \text{GeV}.$$

$$\Gamma(B_s \to \gamma \mu^+ \mu^-) = 2.2 \times 10^{-21} \times \left(\frac{f_{B_s}}{0.2\text{GeV}} \right)^2 \text{GeV}. \tag{9}$$

For B_d meson decay, we take $|V_{tb}V_{td}^*| = 0.01$ and $m_d = 0.35$ GeV. The decay widths are then:

$$\Gamma(B_d \to \gamma e^+ e^-) = 3.6 \times 10^{-22} \times \left(\frac{f_{B_d}}{0.2\text{GeV}} \right)^2 \text{GeV},$$

$$\Gamma(B_d \to \gamma \mu^+ \mu^-) = 2.7 \times 10^{-22} \times \left(\frac{f_{B_d}}{0.2\text{GeV}} \right)^2 \text{GeV}. \tag{10}$$

If the lifetimes are taken as $\tau(B_s) = 1.34 \times 10^{-12}s$, $\tau(B_d) = 1.50 \times 10^{-12}s$ [10], and if $f_{B_q} = 200\text{MeV}$ is used, they correspond to the branching ratios:

$$B(B_s \to e^+ e^- \gamma) = 6.2 \times 10^{-9},$$

$$B(B_s \to \mu^+ \mu^- \gamma) = 4.6 \times 10^{-9}, \tag{11}$$

$$B(B_d \to e^+ e^- \gamma) = 8.2 \times 10^{-10},$$

$$B(B_d \to \mu^+ \mu^- \gamma) = 6.2 \times 10^{-10}.$$

Note that the branching ratios we get for radiative decays are just the same order of the pure leptonic decay $B_q \to \mu^+ \mu^-$. The decay rates for pure leptonic decays are:

$$\Gamma(B_q \to l^+ l^-) = \frac{\alpha^2 G_F^2 f_{B_q}^2 m_{B_q} m_l^2}{16\pi^3} |V_{tb}V_{tq}^*|^2 C_{10}^2. \tag{12}$$

Using the same parameters as in the radiative decay, the numerical results are:

$$
\begin{aligned}
B(B_s \to e^+e^-) &= 6.1 \times 10^{-14}, \\
B(B_s \to \mu^+\mu^-) &= 2.6 \times 10^{-9}, \\
B(B_d \to e^+e^-) &= 4.2 \times 10^{-15}, \\
B(B_d \to \mu^+\mu^-) &= 1.8 \times 10^{-10}.
\end{aligned}
\tag{13}
$$

So for the e^+e^- channel, the radiative decay dominates over the dileptonic one, while for the $\mu^+\mu^-$ channel, the branching ratio of the radiative decay is a little larger than that for the pure leptonic decay.

3 Conclusions

We predict the branching ratios in the SM for $B_s \to \gamma l^+ l^-$ to be around 5×10^{-9} for $B_s \to \gamma\mu^+\mu^-$ and around 6×10^{-10} for $B_d \to \gamma\mu^+\mu^-$, with slightly larger values for $B_s(B_d) \to \gamma e^+e^-$. With these predictions, they will hopefully be detected at LHC-B. thus provide alternative channels for measuring f_{B_s}/m_s (f_{B_d}/m_d). In LHC-B, approximately 6×10^{11} (2×10^{11}) B_d (B_s) mesons are expected per year, therefore there is a good chance of observing the decays considered here.

We conclude with comments on the experimental extractions of the decay constants. In the measurements of the pure leptonic decays, processes with additional soft photons emitted (say, photons with energy less than 50 MeV) might contribute due to the inability of the detector to separate them. From our calculation, we find that these soft photon processes are not important in the radiative dileptonic decays, see Fig.3. Since the efficiency of detecting photons may be low, it will be useful for experiments to have the sum of the radiative decay and the pure leptonic processes:

$$
\begin{aligned}
B(B_s \to \mu^+\mu^- + \gamma\mu^+\mu^-) &= 7.3 \times 10^{-9}, \\
B(B_d \to \mu^+\mu^- + \gamma\mu^+\mu^-) &= 8.1 \times 10^{-10}.
\end{aligned}
\tag{14}
$$

The total $\mu^+\mu^-(\gamma)$ branching ratios for B_s and B_d decay are almost the same as that of the $e^+e^-\gamma$ case.

Acknowledgement

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Figure Captions

Fig.1 Feynman diagrams in standard model for $b\bar{q} \rightarrow l^+l^-$.

Fig.2 Additional diagrams which contribute to $B_s \rightarrow \gamma l^+l^-$. The relevant penguin operators are O_7 (black dot).

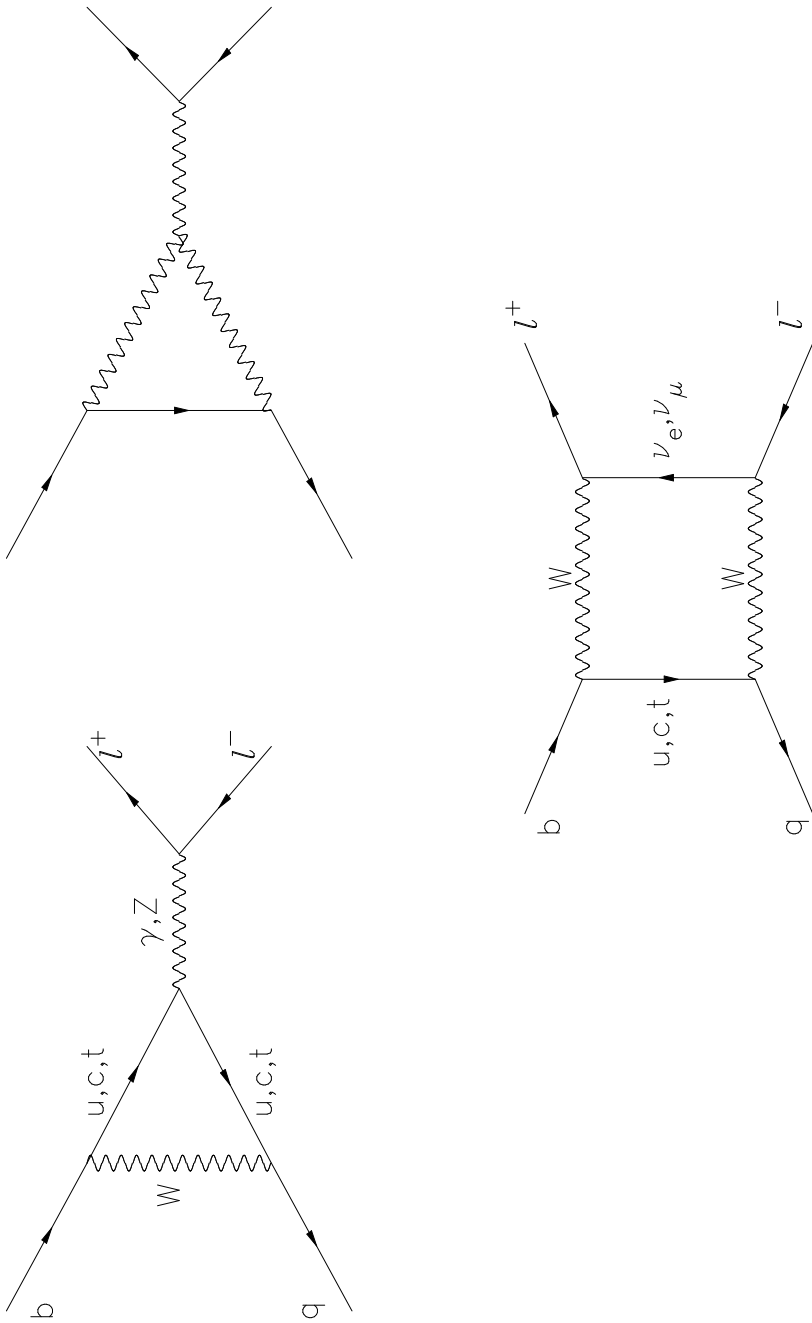Fig.3 Differential decay rates of $B_s \rightarrow \gamma l^+l^-$ versus $s = (p_+ + p_-)^2/m_{B_s}^2$.
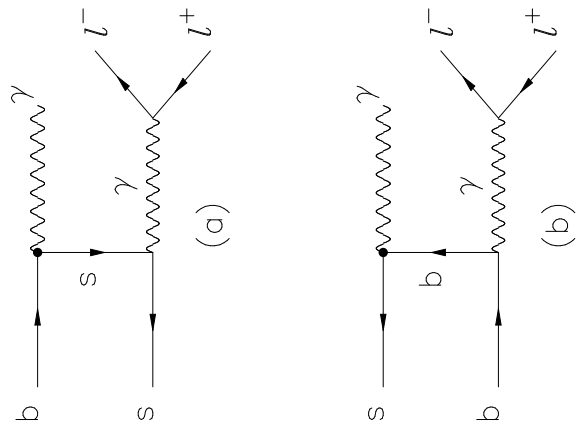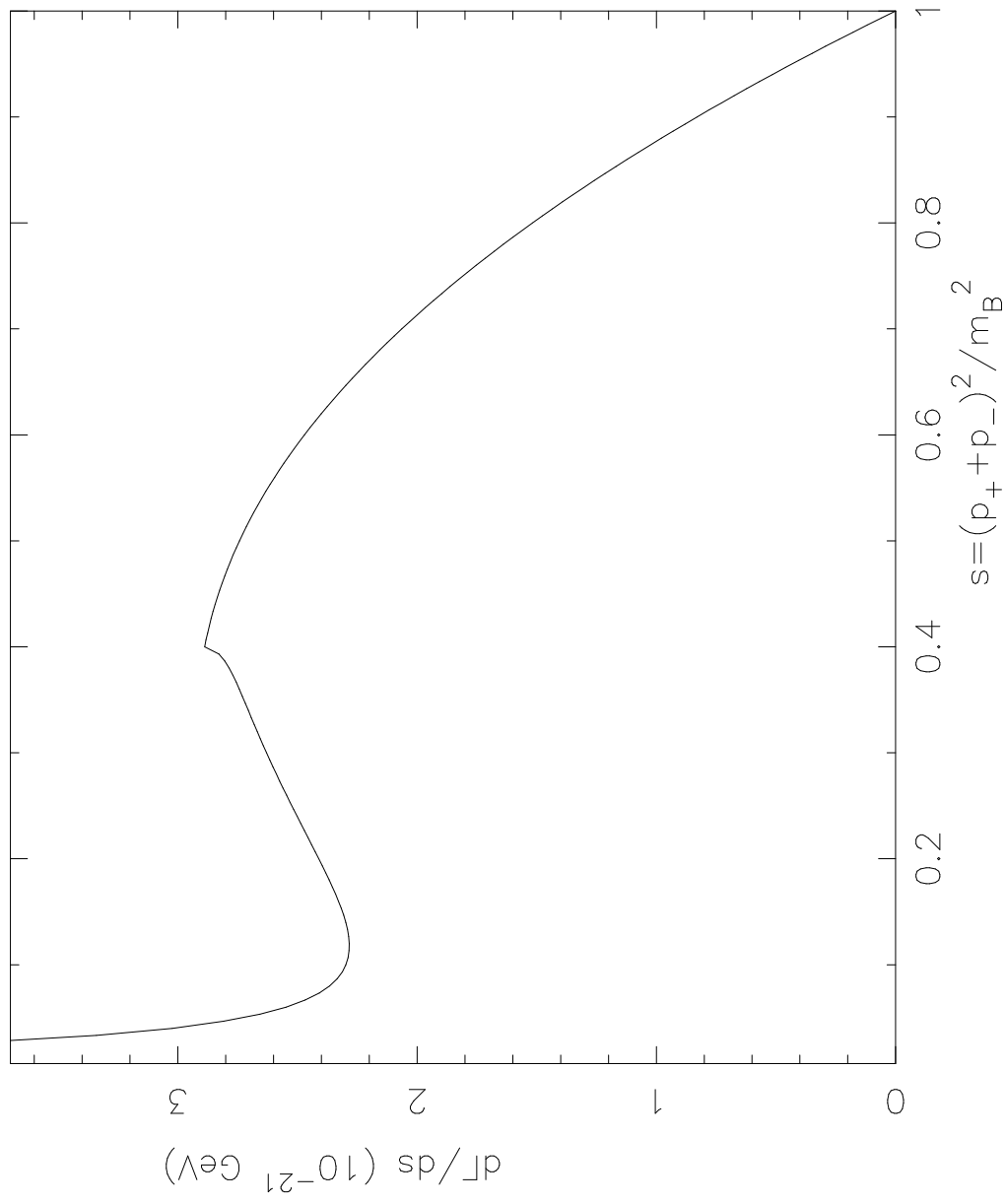
Fig. 1

Fig. 2

$s=(p_+ + p_-)^2/m_B^2$

Fig. 3

$d\Gamma/ds$ (10^{-21} GeV)